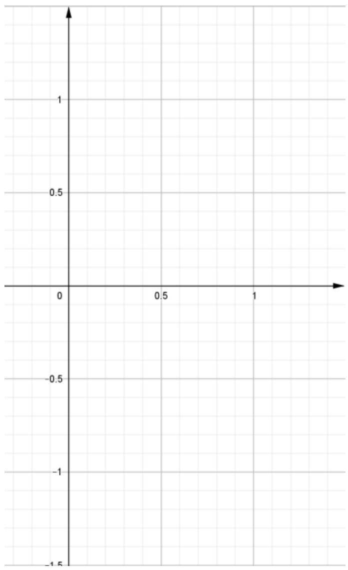
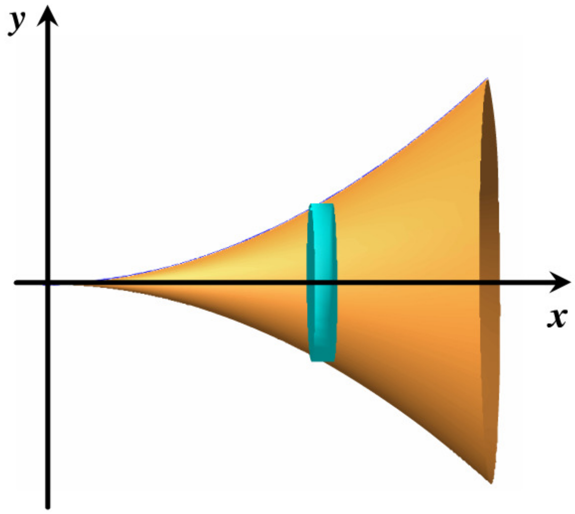
Find the volume of the solid obtained by the rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

1) $y = x^2, \quad x = 1, \quad y = 0$ | about the x-axis

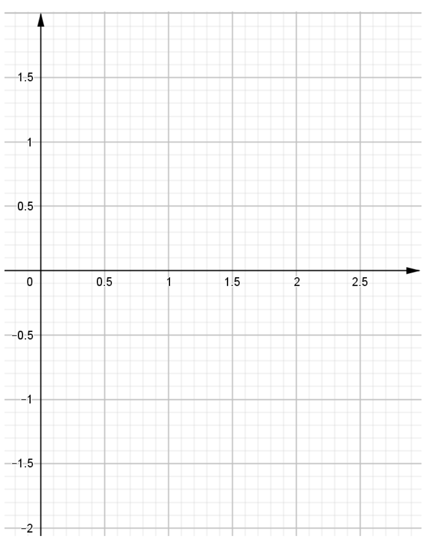
$\boxed{\frac{\pi}{5}}$

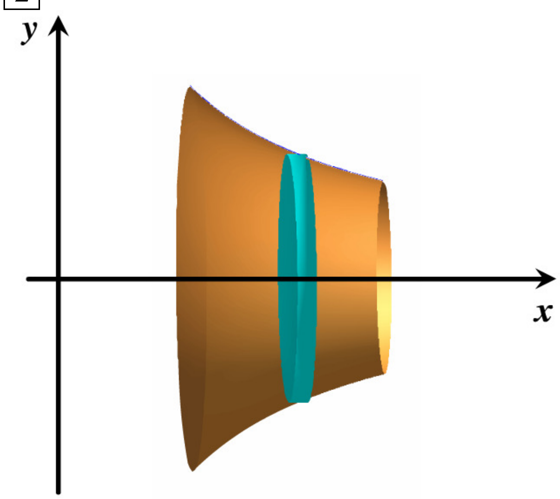




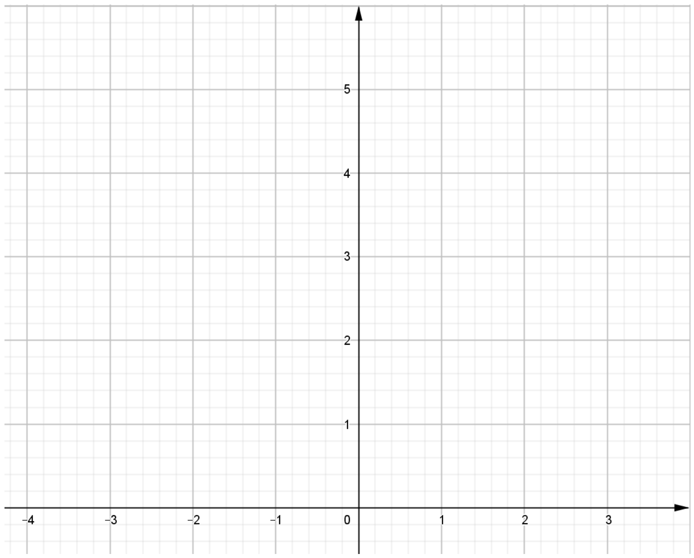
2) $y = \frac{1}{x}, \quad x = 1, \quad x = 2, \quad y = 0$ | about the x-axis

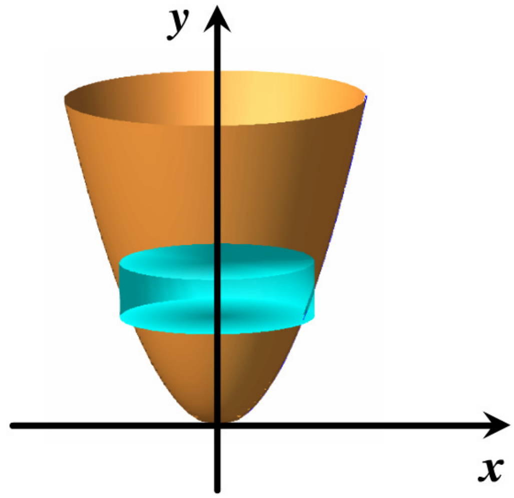
$\boxed{\frac{\pi}{2}}$



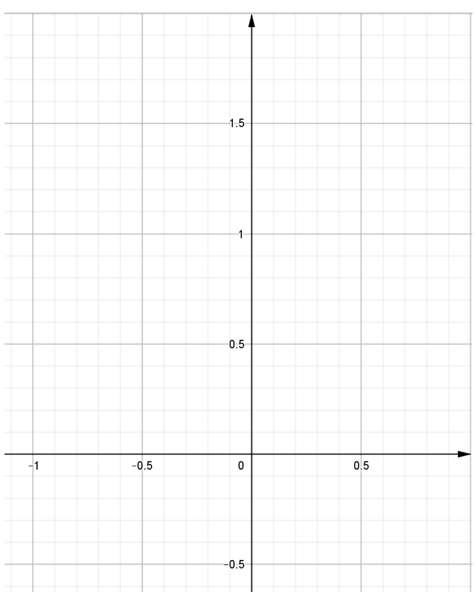


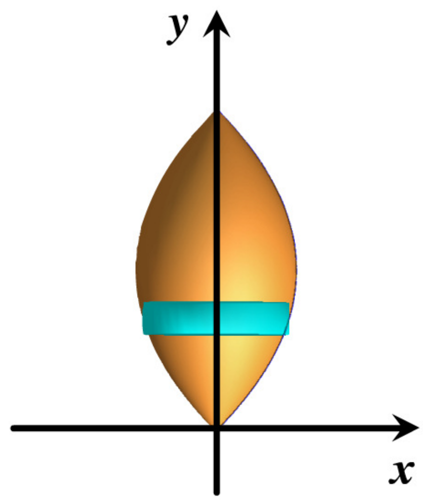
3) $y = x^2, \quad 0 \le x \le 2, \quad y = 4, \quad x = 0$ | about the y-axis $\boxed{8\pi}$





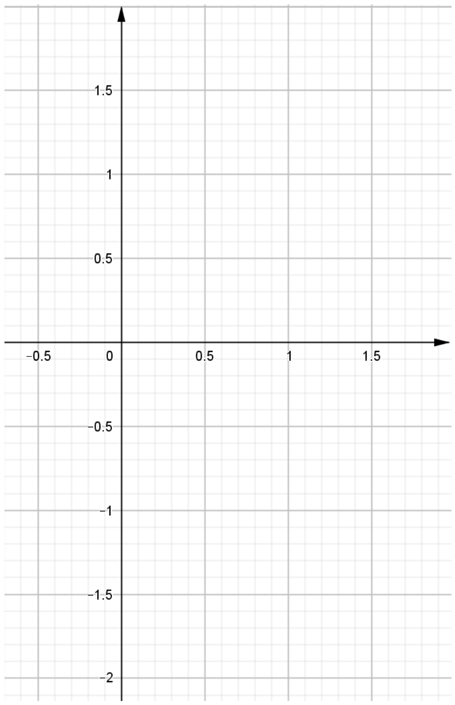
4) $x = y - y^2, \quad x = 0$ | about the y-axis $\boxed{\dfrac{\pi}{30}}$

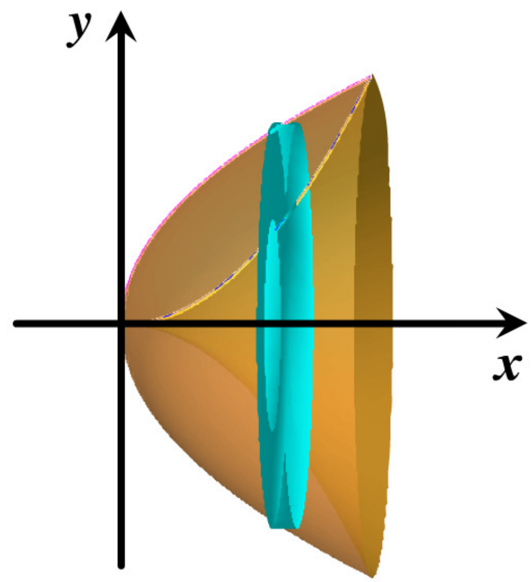




5) $y = x^2, \quad y^2 = x$ | about the x-axis

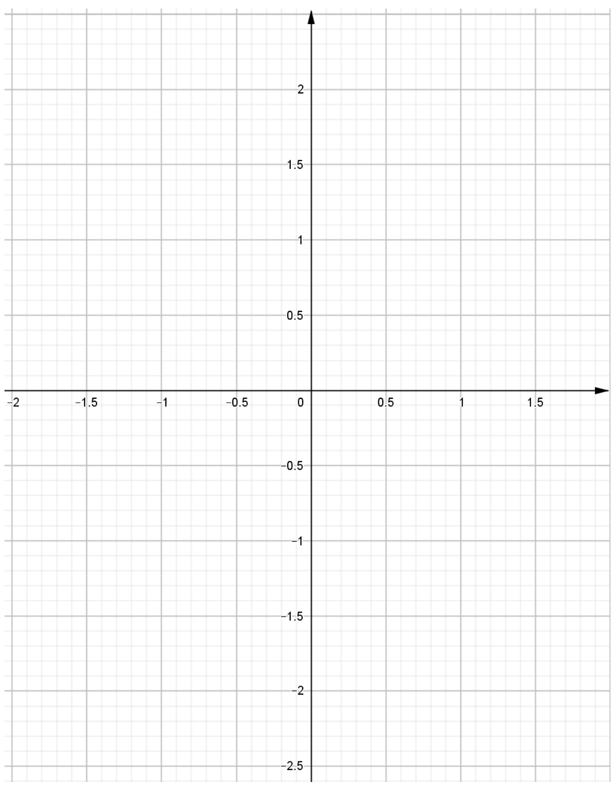
$\boxed{\dfrac{3\pi}{10}}$

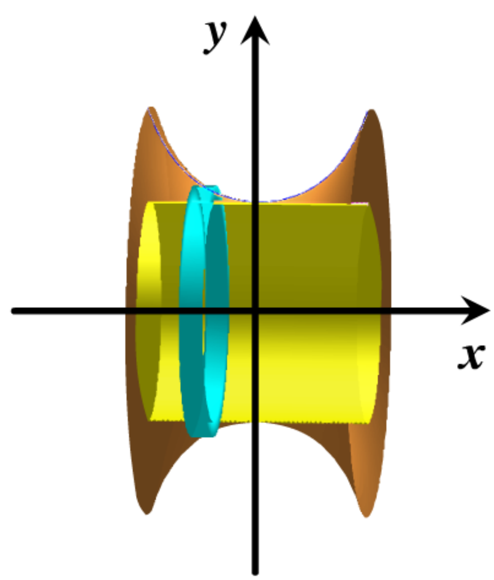




6) $y = \sec x, \quad y = 1, \quad x = -1, \quad x = 1$ | about the x-axis

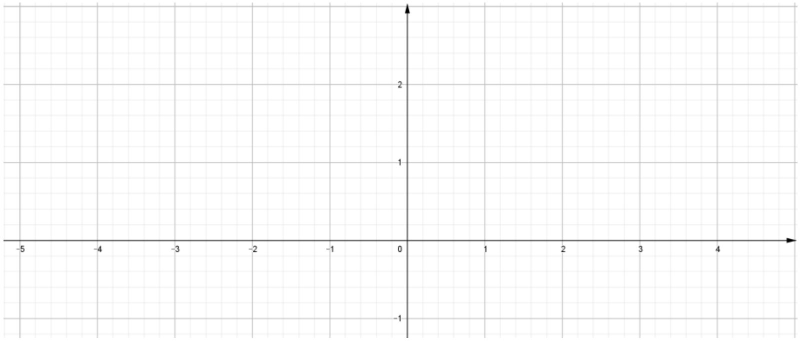
$\boxed{2\pi\,[\tan(1) - 1]}$

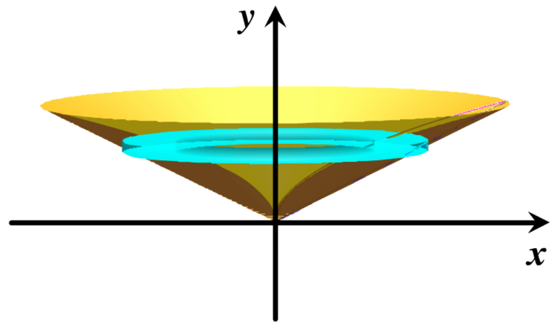




7) $y^2 = x, \quad x = 2y \ \mid$ about the y-axis

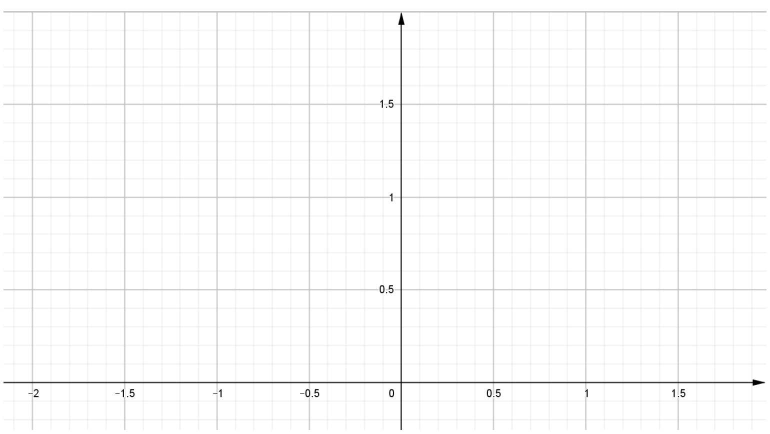
$\boxed{\dfrac{64\pi}{15}}$

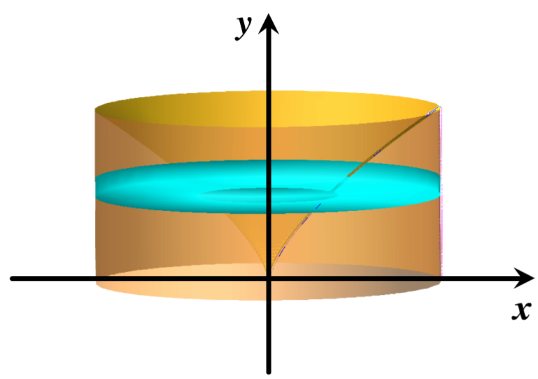




8) $y = x^{\frac{2}{3}}, \quad x = 1, \quad y = 0 \ \mid$ about the y-axis

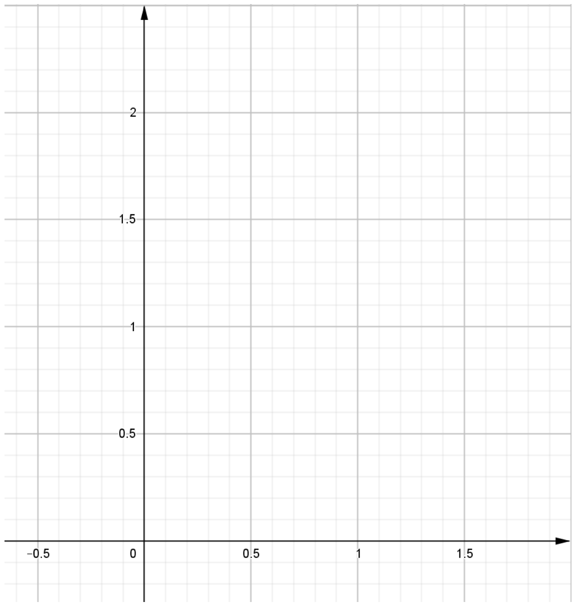
$\boxed{\dfrac{3}{4}\pi}$

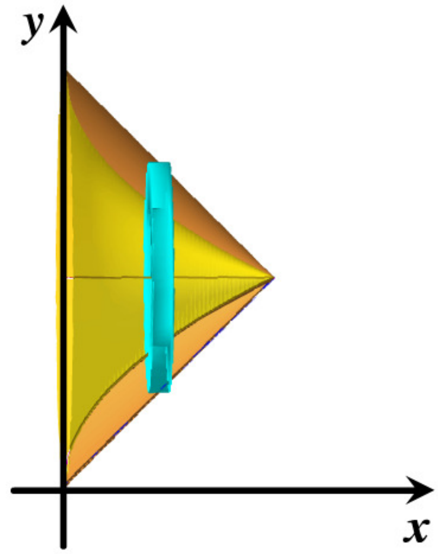




9) $y = x, \quad y = \sqrt{x}$ | about $y = 1$

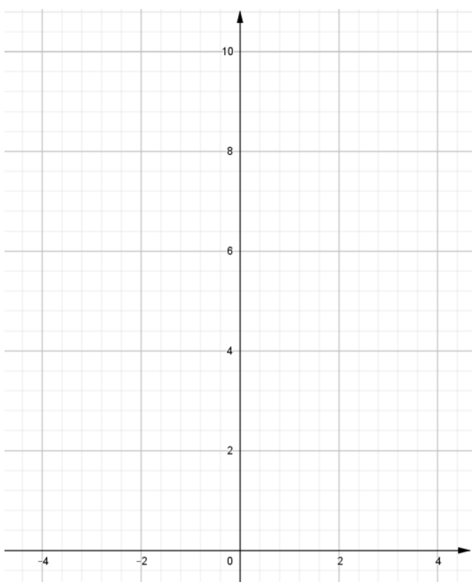
$\boxed{\dfrac{\pi}{6}}$

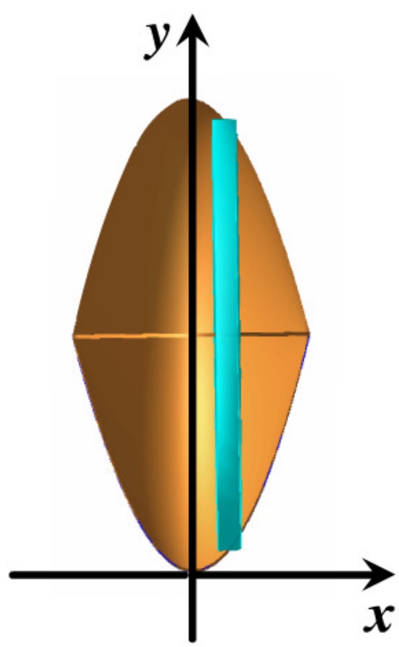




10) $y = x^2, \quad y = 4$ | about $y = 4$

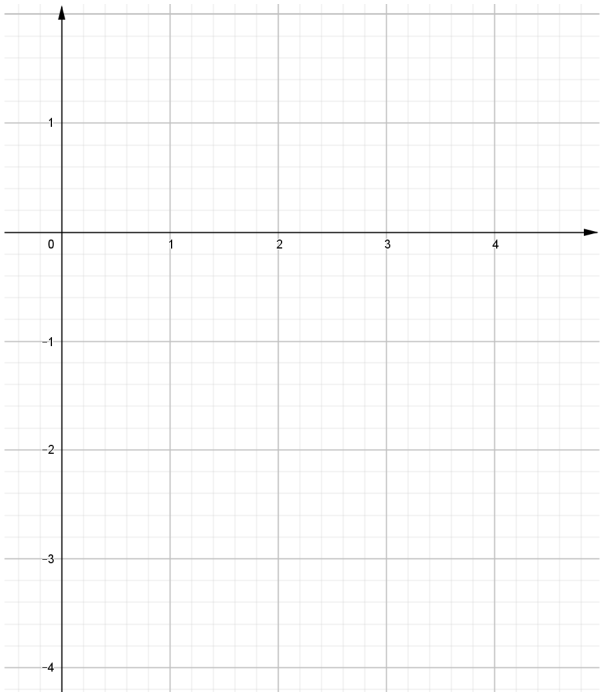
$\boxed{\dfrac{512\pi}{15}}$

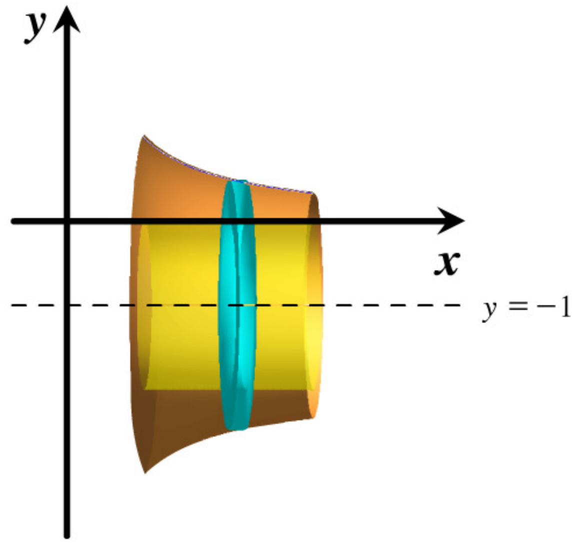




11) $y=\frac{1}{x}, \quad y=0, \quad x=1, \quad x=3$ | about $y=-1$

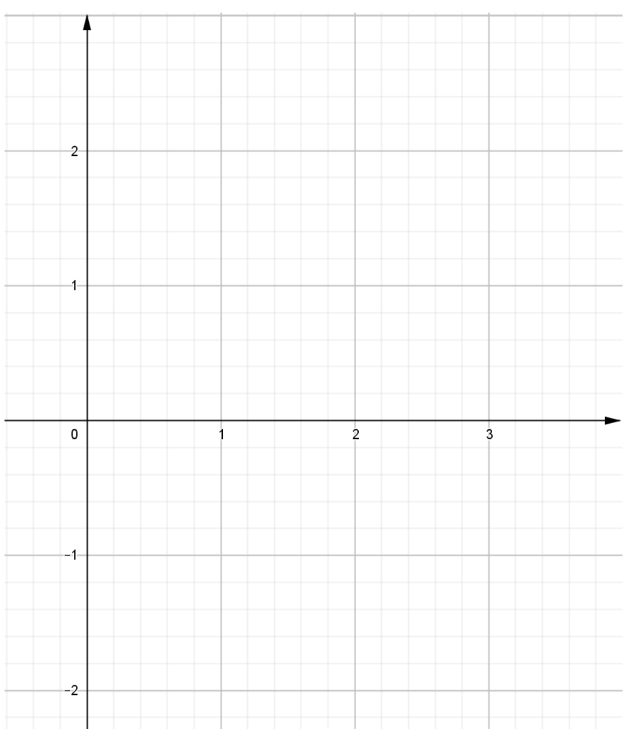
$\boxed{2\pi\left[\ln(3)+\frac{1}{3}\right]}$

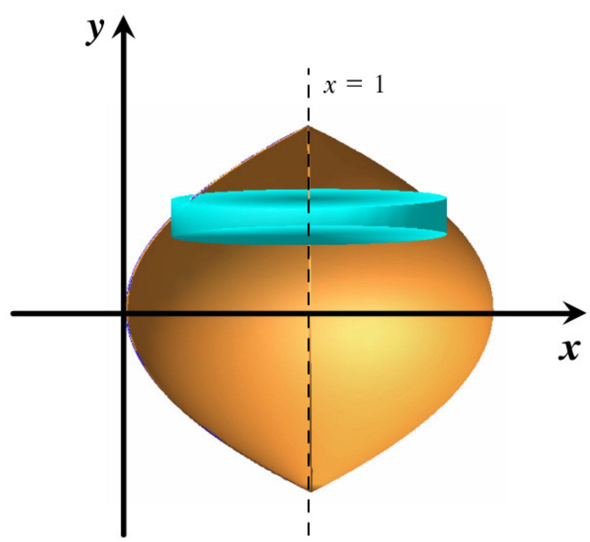




12) $x=y^2, \quad x=1$ | about $x=1$

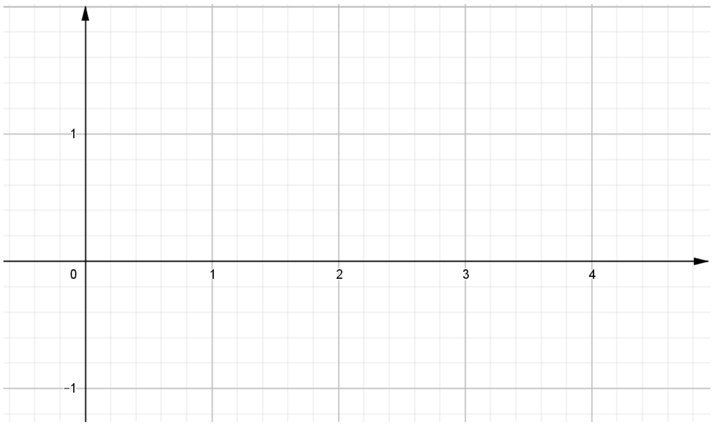
$\boxed{\frac{16}{15}\pi}$

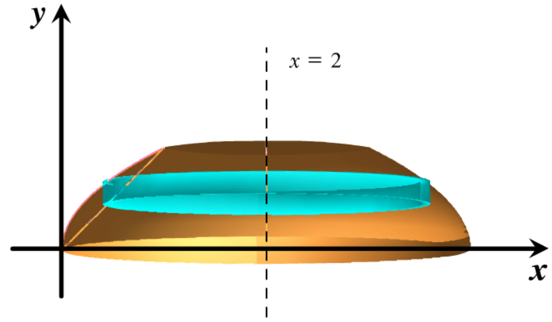




13) $y = x, \quad y = \sqrt{x}$ | about $x = 2$

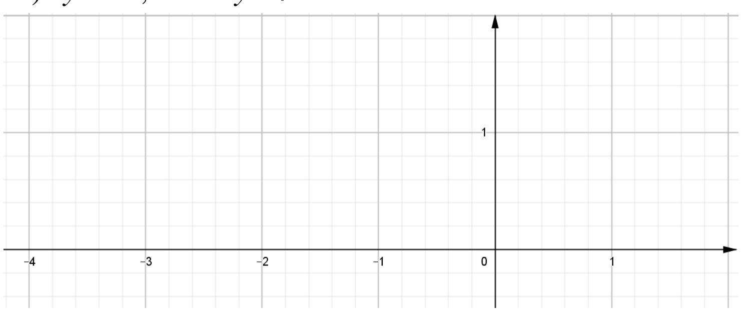
$\boxed{\frac{8}{15}\pi}$

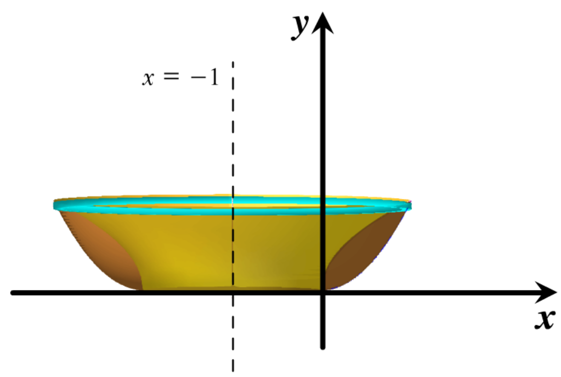




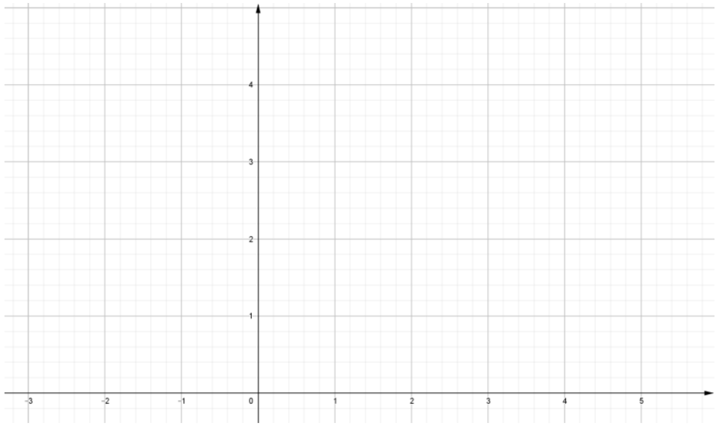
14) $y = x^2, \quad x = y^2$ | about $x = -1$

$\boxed{\frac{29}{30}\pi}$

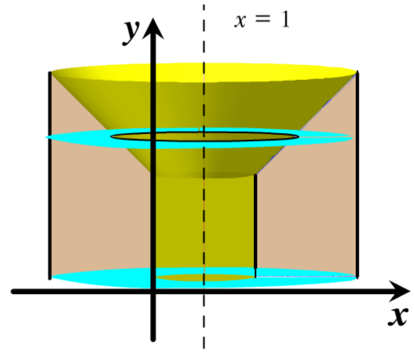




15) $y = x, \quad y = 0, \quad x = 2, \quad x = 4 \mid \text{about } x = 1$



$\boxed{\frac{76}{3}\pi}$



Set up, but do not evaluate, and integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

16) $y = \tan^3 x, \quad y = 1, \quad x = 0 \mid \text{about } y = 1$

$$\boxed{V = \pi \int_0^{\frac{\pi}{4}} \left(1 - \tan^3 x\right)^2 \, dx}$$

17) $y = (x-2)^4, \quad 8x - y = 16 \mid \text{about } x = 10$

$$\boxed{V = \pi \int_0^{16} \left\{ \left[10 - \left(\tfrac{1}{8} y + 2\right)\right]^2 - \left[10 - \left(2 + \sqrt[4]{y}\right)\right]^2 \right\} dy}$$

18) $y=0, \quad y=\sin x, \quad 0\le x\le \pi$ | about $y=-2$

$$\boxed{V=\int_0^{\pi}\left[\left(\sin x+2\right)^2-2^2\right]dx}$$

19) $x^2-y^2=1, \quad x=3$ | about $x=-2$

$$\boxed{V=\pi\int_{-2\sqrt{2}}^{2\sqrt{2}}\left[5^2-\left(\sqrt{1+y^2}+2\right)^2\right]dy}$$

20) Use a graph to find approximate x-coordinates of the points of intersection of the given curves. Then find (approximately) the volume of the solid obtained by rotating about the x-axis the region bounded by these curves.

$$y=3\sin(x^2), \quad y=e^{\frac{x}{2}}+e^{-2x}$$

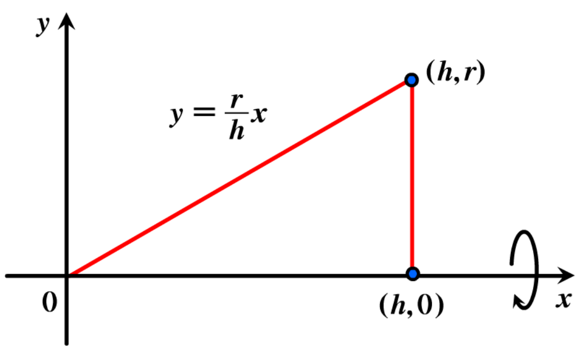
$$\boxed{V\approx 7.519}$$

21) A CAT scan produces equally spaced cross-sectional views of a human organ that provide information about the organ otherwise obtained only by surgery. Suppose that a CAT scan of a human liver shows cross-sections spaced 1.5 cm apart. The liver is 15 cm long and the cross-sectional areas, in square centimeters, are 0, 18, 58, 79, 94, 106, 117, 128, 63, 39, and 0. Use the Midpoint Rule to estimate the volume of the liver.

$$\boxed{V \approx 1110 \text{ cm}^3}$$

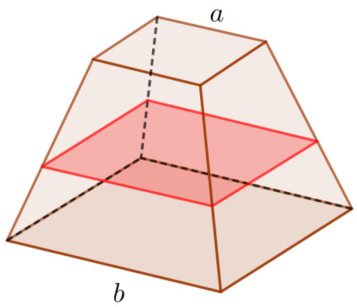
Find the volume of the described solid S.

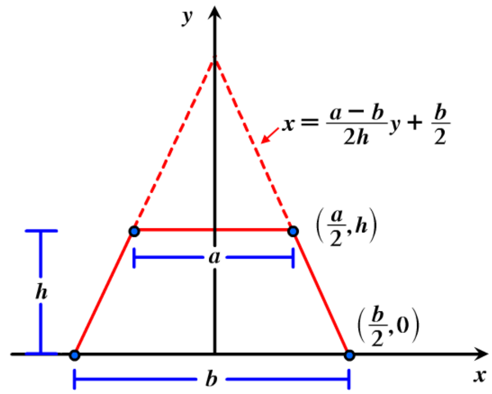
22) A right circular cone with height h and base radius r. Use the following diagram to find the volume by using calculus.



$$\boxed{V = \frac{1}{3}\pi r^2 h}$$

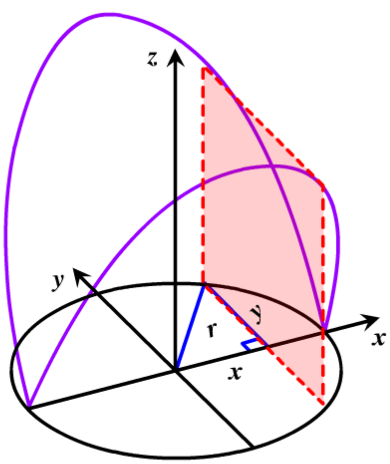
23) A frustum of a pyramid with square base of side b, square top of side a, and height h. What happens if $a = b$? What happens if $a = 0$? Use the following diagram to find the volume by using calculus.





$$V = \frac{1}{3}(a^2 + ab + b^2)h$$
$$a = b \quad \rightarrow \quad V = b^2 h$$
$$a = 0 \quad \rightarrow \quad V = \frac{1}{3}b^2 h$$

24) The base S is a circular disk with radius r. Parallel cross-sections perpendicular to the base are squares. Use the following diagram to find the volume by using calculus.



$$V = \frac{16}{3}r^3$$

25) The base of S is an elliptical region with boundary curve $9x^2 + 4y^2 = 36$. Cross-sections perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base.

$$\boxed{V = 24}$$

26) The base of S is the parabolic region $\left\{(x, y) \mid x^2 \le y \le 1\right\}$. Cross-sections perpendicular to the y-axis are equilateral triangles.

$$\boxed{V = \frac{\sqrt{3}}{2}}$$

27) Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere.

$$\boxed{\frac{5}{24}\pi r^3}$$