

Power Series for Elementary Functions	Interval of Convergence
$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \dots$	$(-1, 1)$
$\ln x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-1)^n}{n} = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} + \dots$	$(0, 2]$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$	$(-\infty, \infty)$
$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$	$(-\infty, \infty)$
$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$	$(-\infty, \infty)$
$\tan^{-1} x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$	$[-1, 1]$
$\sin^{-1} x = \sum_{n=0}^{\infty} \frac{(2n)! x^{2n+1}}{(2^n n!)^2 (2n+1)} = x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3 x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5 x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \dots$	$[-1, 1]$

Taylor's Theorem

If a function f is differentiable through order $n+1$ in an interval I containing c , then, for each x , in I , there exists z between x and c such that

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \dots + \frac{f^{(n)}(c)}{n!}(x-c)^n + R_n(x)$$

Where $R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$.

Lagrange Form of the Remainder

$$|R_n(x)| \leq \frac{|x-c|^{n+1}}{(n+1)!} \max |f^{(n+1)}(z)|$$

Where $|f^{(n+1)}(z)|$ is the maximum value of $f^{(n+1)}(z)$ between x and c .