Find a power series representation for the function and determine the interval of convergence.

1) $f(x) = \dfrac{1}{1+x}$

2) $f(x) = \dfrac{3}{1-x^4}$

3) $f(x) = \dfrac{1}{1+9x^2}$

4) $f(x)=\dfrac{1}{x-5}$

5) $f(x)=\dfrac{x}{4x+1}$

6) $f(x)=\dfrac{x}{9+x^2}$

Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

7) $f(x) = \dfrac{3}{x^2 + x - 2}$

8) For the function: $f(x) = \dfrac{1}{(1+x)^2}$

a) Use differentiation to find a power series representation for the function and find the radius of convergence.

b) Use part a) to find a power series for: $f(x) = \dfrac{1}{(1+x)^3}$

c) Use part b) to find a power series for: $f(x) = \dfrac{x^2}{(1+x)^3}$

Find a power series representation for the function and determine the radius of convergence.

9) $f(x) = \ln(5 - x)$

10) $f(x) = \dfrac{x^2}{(1-2x)^2}$

11) $f(x) = \arctan(x/3)$

Evaluate the indefinite integral as a power series. What is the radius of convergence?

12) $\int \frac{t}{1-t^8}\,dt$

13) $\int \frac{x-\tan^{-1} x}{x^3}\,dx$

14) Show that the function $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$ is a solution of the differential equation $f''(x)+f(x)=0$.

15) Let $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n^2}$ find the intervals of convergence for f , f' , and f''

16) Given the geometric series $\sum_{n=0}^{\infty} x^n$ find the following:

a) The sum of the series:

$$\sum_{n=1}^{\infty} nx^{n-1} \qquad |x| < 1$$

b) The sum of each of the following series:

$$\sum_{n=1}^{\infty} nx^{n} \qquad |x| < 1 \qquad\qquad \sum_{n=1}^{\infty} \frac{n}{2^n}$$

c) The sum of each of the following series

$$\sum_{n=2}^{\infty} n(n-1)x^{n} \qquad |x| < 1 \qquad\qquad \sum_{n=2}^{\infty} \frac{n^2 - n}{2^n} \qquad\qquad \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

17) Use the power series for $\tan^{-1} x$ to prove the following expression for π as the sum of an infinite series:

$$\pi = 2\sqrt{3} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)3^n}$$