Find a power series representation for the function and determine the interval of convergence.

1) $f(x)=\dfrac{1}{1+x}$

$$\boxed{\sum_{n=0}^{\infty}(-1)^n x^n, \quad I=(-1,\ 1)}$$

2) $f(x)=\dfrac{3}{1-x^4}$

$$\boxed{\sum_{n=0}^{\infty}3x^{4n}, \quad I=(-1,\ 1)}$$

3) $f(x)=\dfrac{1}{1+9x^2}$

$$\boxed{\sum_{n=0}^{\infty}(-1)^n 3^{2n} x^{2n}, \quad I=\left(-\frac{1}{3},\ \frac{1}{3}\right)}$$

4) $f(x)=\dfrac{1}{x-5}$ $\quad$ $\boxed{-\sum_{n=0}^{\infty}\frac{1}{5^{n+1}}x^n,\quad I=(-5,\,5)}$

5) $f(x)=\dfrac{x}{4x+1}$ $\quad$ $\boxed{\sum_{n=0}^{\infty}(-1)^n 2^{2n}x^{n+1},\quad I=\left(-\frac{1}{4},\,\frac{1}{4}\right)}$

6) $f(x)=\dfrac{x}{9+x^2}$ $\quad$ $\boxed{\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{9^{n+1}},\quad I=(-3,\,3)}$

Express the function as the sum of a power series by first using partial fractions. Find the interval of convergence.

7) $f(x)=\dfrac{3}{x^2+x-2}$

$$\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}}{2^{n+1}}-1\right]x^n, \quad I=(-1,\,1)$$

8) For the function: $f(x)=\dfrac{1}{(1+x)^2}$

a) Use differentiation to find a power series representation for the function and find the radius of convergence.

b) Use part a) to find a power series for: $f(x)=\dfrac{1}{(1+x)^3}$

c) Use part b) to find a power series for: $f(x)=\dfrac{x^2}{(1+x)^3}$

a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n(n+1)x^n, \quad R=1$

b) $\displaystyle\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n(n+2)(n+1)x^n, \quad R=1$

c) $\displaystyle\frac{1}{2}\sum_{n=2}^{\infty}(-1)^n n(n-1)x^n, \quad R=1$

Find a power series representation for the function and determine the radius of convergence.

9) $f(x) = \ln(5 - x)$

$$\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}, \quad R = 5$$

10) $f(x) = \dfrac{x^2}{(1-2x)^2}$

$$\sum_{n=2}^{\infty} 2^{n-2}(n-1)x^n, \quad R = \frac{1}{2}$$

11) $f(x) = \arctan(x/3)$

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{3^{2n+1}(2n+1)} x^{2n+1}, \quad R = 3$$

Evaluate the indefinite integral as a power series. What is the radius of convergence?

12) $\int \frac{t}{1-t^8}\,dt$ $\boxed{C+\sum_{n=0}^{\infty}\frac{t^{8n+2}}{8n+2}, \quad R=1}$

13) $\int \frac{x-\tan^{-1}x}{x^3}\,dx$ $\boxed{C+\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n-1}}{4n^2-1}, \quad R=1}$

14) Show that the function $f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n)!}$ is a solution of the differential equation $f''(x)+f(x)=0$.

Show

15) Let $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n^2}$ find the intervals of convergence for f, f', and f''

$f \quad I=[-1,1]$

$f' \quad I=[-1,1)$

$f'' \quad I=(-1,1)$

16) Given the geometric series $\sum_{n=0}^{\infty} x^n$ find the following:

a) The sum of the series:

$$\sum_{n=1}^{\infty} nx^{n-1} \quad |x|<1$$

$$\boxed{\frac{1}{(1-x)^2}, \quad |x|<1}$$

b) The sum of each of the following series:

$$\sum_{n=1}^{\infty} nx^n \quad |x|<1$$

$$\boxed{\frac{x}{(1-x)^2}, \quad |x|<1}$$

$$\sum_{n=1}^{\infty} \frac{n}{2^n}$$

$$\boxed{2}$$

c) The sum of each of the following series

$$\sum_{n=2}^{\infty} n(n-1)x^n \quad |x|<1$$

$$\boxed{\frac{2x^2}{(1-x)^3}, \quad |x|<1}$$

$$\sum_{n=2}^{\infty} \frac{n^2-n}{2^n}$$

$$\boxed{4}$$

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

$$\boxed{6}$$

17) Use the power series for $\tan^{-1} x$ to prove the following expression for π as the sum of an infinite series:

$$\pi = 2\sqrt{3}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)3^n}$$

Show