

1) Let  $a_n = \frac{2n}{3n+1}$ .

a) Determine whether  $a_n$  is convergent. Converges

b) Determine whether  $\sum_{n=1}^{\infty} a_n$  is convergent. Diverges

Determine whether the series is convergent or divergent. If it is convergent, find its sum.

2)  $3 + 2 + \frac{4}{3} + \frac{8}{9} + \dots$  Converges, 9

3)  $\sum_{n=1}^{\infty} 5 \left(\frac{2}{3}\right)^{n-1}$  Converges, 15

4)  $\sum_{n=1}^{\infty} \frac{(-6)^{n-1}}{5^{n-1}}$  Diverges

$$5) \sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$$

Converges,  $\frac{3e}{3-e}$

$$6) \sum_{n=1}^{\infty} \frac{n}{n+5}$$

Diverges

$$7) \sum_{n=1}^{\infty} \frac{3}{n}$$

Diverges

$$8) \sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$$

Converges,  $\frac{3}{2}$

$$9) \sum_{n=1}^{\infty} \frac{(n+1)^2}{n(n+2)}$$

Diverges

$$10) \sum_{n=1}^{\infty} \frac{2}{n^2 + 4n + 3}$$

Converges,  $\frac{5}{6}$

$$11) \sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n}$$

Converges,  $\frac{3}{2}$

$$12) \sum_{n=1}^{\infty} \arctan(n)$$

Diverges

$$13) \sum_{n=1}^{\infty} (\cos 1)^n$$

Converges,  $\frac{\cos(1)}{1-\cos(1)}$

$$14) \sum_{n=1}^{\infty} \left( \frac{3}{n(n+3)} + \frac{5}{4^n} \right)$$

Converges,  $\frac{5}{3}$

Express the number as a ratio of integers.

$$15) 0.\overline{2}$$

$\frac{2}{9}$

$$16) 0.\overline{73}$$

$\frac{73}{99}$

17)  $6.\overline{254}$   $\frac{344}{55}$

18)  $0.123\overline{456}$   $\frac{41,111}{333,000}$

Find the values of  $x$  for which the series converges. Find the sum of the series for those values of  $x$ .

19)  $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$   $-3 < x < 3, \quad \frac{x}{3-x}$

20)  $\sum_{n=1}^{\infty} (x-4)^n$   $3 < x < 5, \quad \frac{x-4}{5-x}$

21)  $\sum_{n=0}^{\infty} 4^n x^n$

$$\boxed{|x| < \frac{1}{4}, \quad \frac{1}{1-4x}}$$

22)  $\sum_{n=0}^{\infty} \frac{\cos^n x}{2^n}$

$$\boxed{\text{all real values of } x, \quad \frac{2}{2-\cos x}}$$

23) If the  $n$ th partial sum of a series  $\sum_{n=1}^{\infty} a_n$  is  $s_n = \frac{n-1}{n+1}$  find  $a_n$  and  $\sum_{n=1}^{\infty} a_n$ .

$$\boxed{a_n = \frac{2}{n(n+1)}, \quad \sum_{n=1}^{\infty} a_n = 1}$$

24) What is the value of  $c$  if  $\sum_{n=2}^{\infty} (1+c)^{-n} = 2$ ?

$$\boxed{c = \frac{\sqrt{3}-1}{2}}$$