Find the Maclaurin series for $f(x)$ using the definition of a Maclaurin series. Assume that $f$ has a power series expansion. Do not show that $R_n(x) \to 0$. Also find the associated radius of convergence.

1) $f(x) = \cos x$

2) $f(x) = \sin 2x$

3) $f(x) = (1+x)^{-3}$

4) $f(x) = xe^x$

Find the Taylor series for $f(x)$ centered at the given value of $a$. Assume that $f$ has a power series expansion. Do not show that $R_n(x) \to 0$. Also find the associated radius of convergence.

5) $f(x) = 1 + x + x^2, \quad a = 2$

6) $f(x) = e^x, \quad a = 3$

7) $f(x) = \sin x, \quad a = \dfrac{\pi}{2}$

8) $f(x) = \dfrac{1}{\sqrt{x}}, \quad a = 9$

Use a derived Maclaurin series to obtain the Maclaurin series for the given function. Also find the associated radius of convergence.

9) $f(x) = e^{-x/2}$

10) $f(x) = x\tan^{-1} x$

11) $f(x) = x\cos 2x$

12) $f(x) = \sin^2 x$ [Hint: Use $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$.]

13) $f(x) = \begin{cases} \dfrac{x - \sin x}{x^3} & \text{if } x \neq 0 \\ \dfrac{1}{6} & \text{if } x = 0 \end{cases}$

Evaluate the indefinite integral as an infinite series.

14) $\int x \cos(x^3)\, dx$

15) $\int \dfrac{\sin x}{x}\, dx$

16) $\int \frac{e^x - 1}{x}\, dx$

Use series to evaluate the limit.

17) $\lim_{x \to 0} \frac{x - \tan^{-1} x}{x^3}$

18) $\lim_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$

Use multiplication or division of power series to find the first three nonzero terms in the Maclaurin series for each function.

19) $y = e^{-x^2}\cos x$

20) $y = \dfrac{x}{\sin x}$

Find the sum of the series.

21) $\sum_{n=0}^{\infty}(-1)^n \dfrac{x^{4n}}{n!}$

22) $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$

23) $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \cdots$